AN OPTIMISATION METHODOLOGY FOR MINING PORTFOLIO MANAGEMENT

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ABSTRACT
Capital investment decisions are usually accomplished by calculating net present values (NPV), often qualified by sensitivity analysis. Conventional wisdom dictates that projects with positive NPV should be funded. However, almost all mining organisations are capital constrained and they must select only those mining projects (exploration opportunities) that best fit the organisations capital structure, operating cash flow position, strategic considerations and financial expectations. Such portfolio decisions tend to be made using simple guidelines that ignore the inherent financial risk (eg by rank and cut), or subjectively based on the manager's experience. As a result, the goal of evaluating investments effectively and accurately accounting for tradeoffs between financial risk and potential return has remained incompletely realised.

Portfolio analysis tools can simultaneously consider cognitatively complex portfolio aspects such as financial return goals, catastrophic risk avoidance, partial participation, optimal investment timing, and performance probability under capital, managerial, strategic and project constraints. Furthermore, project specific uncertainties related to mineral grade, commodity prices, operating costs, and exploration success can also be built into these analysis tools.

This paper proposes an integration of Monte Carlo-based simulation and heuristic optimisation techniques into a global system to provide alternate mining portfolios that represent the best possible trade-off between financial risk and return under various constraints. Practical implementation of this new approach to mining portfolio optimisation is discussed.

INTRODUCTION
In the mining industry investment opportunities tend to be evaluated by assessing the most likely values for essential project parameters to forecast a deterministic cash flow over the life of the investment. With this method, the investment's worth or net present value (NPV) equals these cash flows discounted to present value, typically using the weighted average cost of capital. A positive (negative) NPV creates (destroys) value for the company. This framework provides managers with a simple rule for investment decision making:

$$\text{NPV} > 0 = \text{Accept};\ \ \text{NPV} < 0 = \text{Reject}$$

When a company is capital constrained and cannot fund all of its investment opportunities, projects can be ranked by some profitability index (such as the profit to investment ratio), and funded up to the capital constraint. This is known as the rank and cut approach to capital investment. Rank and cut maximises value creation per investment dollar and provides the company with the maximum expected value creation for a given capital budget. However, rank and cut ignores three fundamental investment issues that may lead to financial under-performance:

1. most project parameters used in NPV calculations are uncertain;
2. capital may not be the sole corporate constraint or goal; and
3. significant asset dependencies may exist that could lead to increased exposure to potentially diversifiable risk. For example, if all the mining assets are copper mines, then there is a high degree of risk related to falls in the copper price. However, if the assets include mines involving multiple commodities, then the risk related to falls in the copper price may be partially offset by rises (or less pronounced falls) in the other commodity prices.

Calculation of the NPV for a given mine/mineral project (hereafter referred to as an asset) relies on estimates of numerous parameters, including:

- the mineral grades;
- the extraction sequence and timing;
- the mineral recovery;
- the commodity price;
- the capital and operating costs; and
- in the case of exploration projects, the probability of eventual development.

All of these parameters are uncertain and should be modelled stochastically. For example, mineral grade values by geostatistical simulations, operating costs with growth functions, and commodity prices using periodic models. The cumulative distribution of total financial payoffs F_x for an asset x can be derived from the combination of a series of stochastic models of mineral grades, costs, prices, and recoveries.

In practice, mining companies typically face asset investment decisions that must satisfy a number of constraints (including capital), financial goals, and complex asset dependencies. The aim of portfolio optimisation is to select a group of assets that jointly maximises a value metric and minimises a risk metric, and satisfies these constraints. In this paper, the value metric is the expected payoff and the risk metric is the downside financial risk.

CONSTRAINED PORTFOLIO OPTIMISATION

Portfolio optimisation problems can be expressed in the following form:

$$\text{maximise} \qquad \lambda E\{F_n\} - (1-\lambda) r\{F_n\} \qquad 0 \le \lambda \le 1 \tag{1}$$

where, λ is a weighting parameter to reflect the risk/reward strategy of the company; $E\{.\}$ is the usual expectation operator; $r\{.\}$ is a real valued risk function; and F_n is the cumulative distribution of total (discounted) financial payoffs for a portfolio of $n \le N$ assets. In a modern portfolio theory context, solutions to Eq. (1) for various λ identify the constrained (discrete) efficient frontier that shows the best possible trade-off between expected payoff and financial risk.

In this paper, F_n is considered to be the equity-proportioned sum of the NPV of each asset chosen for the portfolio, and similarly for annual values of production (p), capital (c). We adopt the downside risk function proposed by Fishburn (1977) as a suitable measure of risk.

Given L potential NPV outcomes for an asset (related to L realisations of grade values, metal prices, etc), we can calculate:

- the NPV for any realisation l, $NPV_l = \sum_{i=1}^{N} npv_{i,l} x_i$ (2)

- the expected NPV for L realisations, $NPV_L = \frac{1}{L}\left\{\sum_{l=1}^{L} NPV_l\right\}$ (3)
- the downside risk, $Risk = \frac{1}{L}\sum_{l=1}^{L}(t - NPV_l)^{\alpha}$, if $NPV_l < t$ (4)

where x_i represents the equity interest (proportion) in the i^{th} asset, t is the corporate NPV target for the mining portfolio, and α (≥0) is a corporate-specific measure of the impact of the NPV failing to reach t.

When complex constraints and financial goals are considered, solutions to Eq. (1) must be solved heuristically. As an example, we describe the efficient frontier optimisation of a portfolio of N assets with annual metric constraints on production (P_k) and capital (C_k) over the next K years as:

$$\text{maximise} \qquad \lambda NPV_L - (1-\lambda) Risk \qquad 0 \le \lambda \le 1 \tag{5}$$

subject to:

$$\sum_{i=1}^{N} p_{i,k} x_i \ge P_k \text{ for } k = 1,\ldots,K \tag{6}$$

$$\sum_{i=1}^{N} c_{i,k} x_i \le C_k \text{ for } k = 1,\ldots,K \tag{7}$$

$$0 \le x_i \le 1 \tag{8}$$

Due to the potential multi-commodity nature of mining portfolios, gross revenue could be used as a production proxy. To satisfy other corporate goals, additional constraints could also be considered, for example (Lessard, 2003):

Minimum/maximum Equity Participation Constraint

The equity constraint (Eq. 8) is replaced with:

$$x_i = 0 \text{ or } x_{\min} \le x_i \le x_{\max} \tag{9}$$

where $x_{\min}$ and $x_{\max}$ represent the minimum and maximum equity participation for any asset. To account for the retention of current assets, these constraints could be made asset specific with $x_{i,\min} = x_{i,\max}. = x_{i,\text{cur}}$, the current asset proportion. Alternatively, the constraint in Eq. (9) would permit partial or total divestment of current assets.

Maximum Project Participation Constraint

Additional constraints could be considered based on management's ability to actively manage a maximum of $n_{\max}^{act}$ projects and/or passively participate in a maximum of $n_{\max}^{pas}$ projects as:

$$\sum_{i=1}^{N} ix_i \le n_{\max} \tag{10}$$

where ix_i is an indicator variable equal to 1 if the asset participation $x_i > 0$, and 0 otherwise.

Optimal Investment Timing

Investment decisions and mine planning cycles typically occur on time scales of months or years. For simplicity, in this paper we will reduce the problem of time to invest into the question of which year to invest, thus formulating essentially a continuous selection constraint into one that is discrete. As a result, the time to invest for each asset can be viewed as a set of new mutually exclusive opportunities, and should be considered as new assets $x_{i,j}$, where j represents the year of investment. For example, $x_{1,1}$, $x_{1,2}$, and $x_{1,3}$ represent the equity choices of the first asset for the next three years. To constrain the problem to a single selection of a unique asset the following expressions are used:

$$\sum_{j=1}^{J} ix_{i,j} \leq 1 \quad \text{, and} \tag{11}$$

$$x_{i,j} - ix_{i,j} \leq 0 \text{ for each } i \text{ and } j. \tag{12}$$

where $ix_{i,j}$ is an indicator variable equal to 1 if $x_{i,j}$ is non-zero, and 0 otherwise.

For delayed investment, the NPV for many potential production assets will typically be reduced unless maximum production happens to coincide with the peak in typically cyclical commodity prices. However, the contribution of individual assets to corporate constraints will also have altered as capital costs will have been shifted into future years. Note that, capital expenditure timing can also be modified implicitly by considering alternate mining plans with a corresponding impact on other key variables such as production and revenue.

APPLICATION TO MINING PORTFOLIOS

Practical application to mining portfolio optimisation under various constraints can be considered for sets of mines and exploration projects. For each mining asset the distribution of potential NPV's can be calculated by considering one or more of the following:

- realisations of grade uncertainty by conditional simulation;
- realisations of commodity prices using a periodic function;
- realisations of operating costs using a growth function;
- potential capital costs drawn randomly from a normal distribution;
- potential mineral recoveries drawn randomly from a normal distribution;
- potential discount rates drawn randomly from a normal distribution; and
- potentially deferring the decision to invest for several years.

For each potential exploration asset the distribution of potential NPV's could also account for:

- the probability of success, i.e. eventual mine development;
- potential development timing in years drawn randomly from a normal distribution;
- potential mining reserves drawn randomly from a normal distribution; and
- potential exploration costs drawn randomly from a normal distribution positively correlated to the reserve tonnage.

Efficient frontiers for any combination of corporate constraints and goals could be generated by a number of heuristic methods, for example, the iterative constructive heuristic (ICH) of Richmond and Beasley (2004). ICH's are ideally suited to constrained portfolio optimisation in that they can search large regions of the solution space, they focus on optimality in the latter stages of the search, and have characteristics similar to repair algorithms to ensure feasibility if achievable. Note that, as the constraints/goals become more onerous, the number of feasible solutions decreases and results in less financially efficient (or dominated) portfolio opportunities. Figure 1 shows an example of efficient frontiers when constraints are progressively increased. In Figure 1, any portfolio located to the right and below an alternate portfolio is dominated by that portfolio. In other words, ideally we would prefer our portfolio to be located in the upper left-hand corner (maximum payoff and zero risk) of Figure 1.

CONCLUSIONS

A framework for optimal investment in portfolios of mining and exploration projects was developed and practical considerations noted. Typical corporate constraints and goals, such as desirable annual production/revenue, limitations on available capital, acceptable equity participation, tolerable project participation, and optimal investment timing, were accounted for.

Consideration of too many constraints/goals may result in failure to find a feasible solution. In such instances, a measure of distance from feasibility could be considered.

Given a fixed portfolio of assets with multiple development options, the portfolio management framework described in this paper could be used to optimise development selection and timing.

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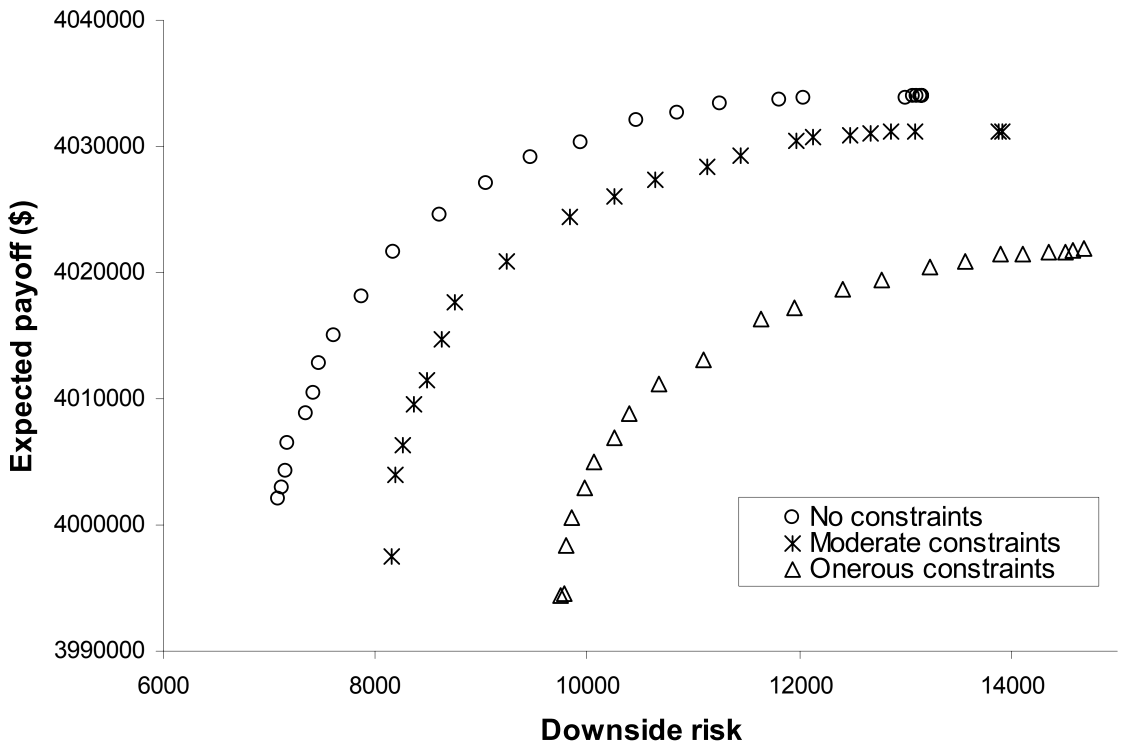

Figure 1 Example efficient frontiers for various constraints.