MULTIVARIATE CONDITIONAL SIMULATION USING SELF-HEALING BASED ON CLOUD DISTRIBUTIONS

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ABSTRACT

There are well established multivariate conditional simulation techniques for correlated variables, but most of these methods require a strong assumption of multivariate Gaussian distribution, can only be applied to a limited number of variables, and cannot deal adequately with complex relationships between multiple variables.

This paper presents a new multivariate conditional simulation method based on local self-healing during sequential Gaussian simulation with the aid of cloud distributions. The technique does not require complex initial data transforms and can easily deal with heteroscedastic and heterotopic distributions as well as inequality constraints. This new approach to stochastic simulation is demonstrated for a polymetallic deposit and a porphyry copper deposit.

INTRODUCTION

Stochastic simulation of multivariate distributions may be desirable for many types of mineral deposits, for example, porphyry copper, iron ore, and nickel laterite mineralisation. Multivariate spatial data from such deposits may exhibit strong correlations between variables, heterotopic distributions, and inequality constraints (Figure 1). Any analysis or modelling must therefore allow for the dependencies that are found in the observed data.

There are a number of well-established conditional simulation methods for multivariate data, of which stepwise conditional transformation (SCT; Leuangthong and Deutsch, 2003) and minimum/maximum autocorrelation factors (MAF; Desbarats and Dimitrakopoulos, 2000), are most commonly used in the mining industry. Both of these techniques require an additional transformation of spatially located Gaussian datasets that have already been transformed independently from the original data space. The second transformation for SCT considers bivariate relationships sequentially to generate zero-lag uncorrelated service variables, whilst MAF considers multivariate relationships to generate service variables that are de-correlated at all lags under certain conditions. However, both SCT and MAF may not work effectively in the presence of heterotopic distributions and inequality constraints. Furthermore, locations not sampled for all variables are precluded from the secondary transformations.

This paper presents a new multivariate conditional simulation method based on local self-healing during sequential Gaussian simulation with the aid of *a priori* cloud distributions. The technique does not require complex initial data transforms and can easily deal with heteroscedastic and heterotopic distributions as well as inequality constraints. This new approach to stochastic simulation is demonstrated for a polymetallic deposit and a porphyry copper deposit.

METHODOLOGY

Multivariate conditional simulation method based on local self-healing during sequential Gaussian simulation with the aid of cloud distributions involves:

1. Independent Gaussian transformation of each original dataset;
2. Calculation of the bivariate/multivariate cloud distributions; and
3. SGS of each variable considering the clouds from step (2), where:
    a. the 1st variable is simulated as normal; and
    b. subsequent variables have the Gaussian distribution from which the simulated value is drawn, adjusted (or healed) to reflect the bivariate relationships identified in Step (2) above.

Step (1) above is the standard approach for independent Sequential Gaussian Simulation (SGS), SCT, or MAF.

For the case studies in this paper the author used a simple probability function based on discretisation of scatterplots for the calculation of the cloud function in Step (2) above. This method preserves strong correlations between variables, heterotopic distributions, and inequality constraints. Alternative approaches such as contouring could be considered.

Step (3) above involves self-healing the SGS (*a priori*) local distribution at a location *x* (that is defined by the conditional mean and variance) by modifying it to account for the cloud functions from Step (2) above. Modification can involve straightforward Bayesian updating, or minimisation of the cross-entropy or some directed divergence measure. The former approach was used by the author for the examples shown in this paper, which involved discretisation of the conditional distribution. As a consequence, the resulting outcomes may vary with:

1. The level of discretisation used to define the cloud function and SGS-derived conditional distribution; and

2. The ordering of the variables if only bivariate cloud functions are considered.

By way of example of Step 3(b) above, consider the cumulative probability plots shown in Figure 2. The thin solid line represents a Gaussian distribution for a node based on a conditional mean of 1.31 and a conditional standard deviation of 0.74 established by SGS for the 2nd variable. The thin dashed line represents the cumulative distribution of the bivariate cloud function for the 2nd variable conditional to the normal score value of 1.35 simulated for the 1st variable, as demonstrated from the scatterplot of normal score in Figure 1. The self-healed cumulative distribution is represented by the thick line in Figure 2. For a random number of 0.95, the normal score value drawn from the traditional SGS would be 1.55, which is slightly above the upper bounds indicated by the data scatterplot in Figure 1. After self-healing, the simulated normal score and for the proposed approach would be 1.46, slightly inside the cloud of points in Figure 1. In simple terms, the proposed approach removes the probability of simulating a value outside the defined cloud and redistributes the initial likelihood of these improbable outcomes to feasible outcomes that fall within the cloud.

The author found that multivariate cloud functions resulted in poor quality solutions as the SGS conditional distributions were significantly modified, resulting in poor reproduction of input variograms. The use of bivariate cloud functions resulted in the implicit reproduction of all cross-correlations, even when all bivariate cloud functions were not considered explicitly.

The key to producing good solutions was to have an adequate level of discretisation of both the cloud function and SGS-derived conditional distribution, and to simulate the variables in decreasing degree of correlation (heterotopicity).

EXAMPLES

Polymetallic Deposit

Figure 3 shows scatterplots of normal score transforms of drill hole data for four variables (Cu, Au, As, and Hg). Cu and Au were economic contributors, whilst As and Hg had potential detrimental environmental consequences and concentrate penalty issues. The deposit had limited drilling that was widely spaced relative to the spatial correlations. Note also that a small number of samples did not have As and Hg assays. The scatterplots in Figure 3 indicate that the variables are correlated to various degrees.

Figure 4 shows scatterplots of the first SGS realisation normal scores when each variable was simulated independently. For these scatterplots, note that:

- there is a very low correlation between all 4 variables that is not consistent with the input drill hole data shown in Figure 3; and

- there is an apparent artefact at a normal score value of 4 in some plots, which is related to a constraint included in the SGS program employed to minimise the potential for extreme values.

Figure 5 shows scatterplots of the first SGS realisation normal scores using the proposed self-healing methodology. For this implementation Au was the primary variable, thus Au-Cu, Au-As, and Au-Hg relationships were considered explicitly during simulation. For these scatterplots, note that the correlations between all 4 variables are in good agreement with the corresponding input drill hole data plots shown in Figure 3. Note also that all data was included in the simulation process, even when As and Hg assays were absent.

Porphyry Copper Deposit

The top plot in Figure 6 shows a scatterplot of total copper (Cu) versus cold sulphuric acid soluble copper (CuCx) for an oxide domain in a porphyry copper deposit. Note that, due to the sequential nature of these assays CuCx must be ≤ Cu, known as an inequality constraint. As a consequence, all data points should fall on or under the 45^o line y=x in Figure 6, as demonstrated by the drill hole data. The deposit was well drilled with 5,287 samples with the domain in question.

The middle plot in Figure 6 shows a scatterplot of Cu versus CuCx for an SGS realisation when each variable was simulated independently. Note that, the inequality constraint is not honoured in this plot as there are significant numbers of points above the 45^o line. Caceres, *et al*. (2011) found similar outcomes when using sequential Gaussian co-simulation and MAF on a similar dataset. The bottom plot in Figure 6 shows a scatterplot of Cu versus CuCx for an SGS realisation using the proposed self-healing methodology where the inequality constraint is honoured.

CONCLUSIONS

A simple method for simulating multiple correlated variables was proposed and demonstrated using SGS on a polymetallic deposit. In this new approach to multivariate simulation, cloud distributions are used to heal conditional distributions prior to the simulated normal score being drawn randomly. The healing process ensures that unrealistic conditional distributions are adjusted to reflect the relationships present in the input data, which does not have to be exhaustively sampled for all variables.

Limited implementation of this new technique has shown great promise. Early issues of low variance simulations and poor reproduction of variograms for secondary variables were noted.

However, the healing approach was adapted to significantly improve these issues. In the case studies to date cross-correlations at distances other than zero have been reasonably honoured implicitly.

REFERENCES

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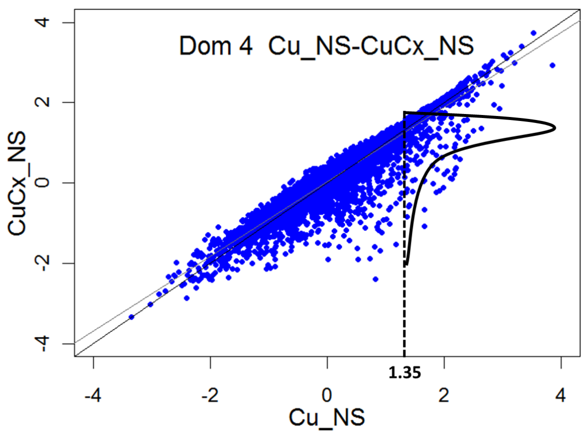

Figure 1. Scatterplot of drill hole data normal scores.

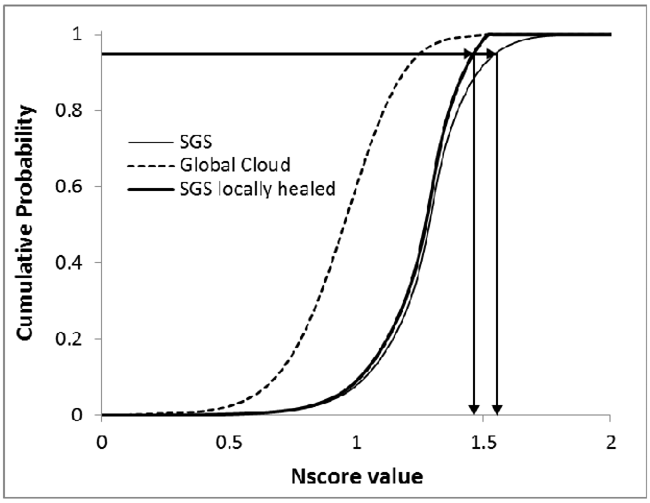

Figure 2. Cumulative probability plots

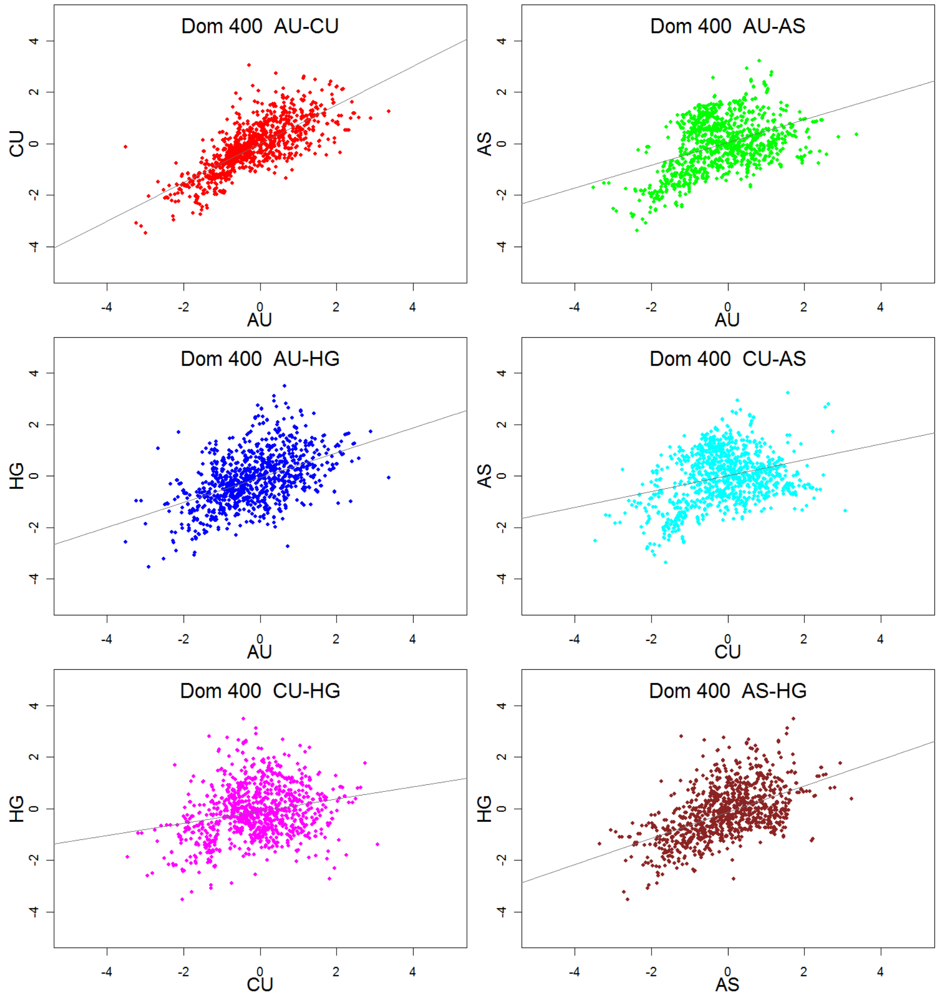

Figure 3. Scatterplots of drill hole data normal scores.

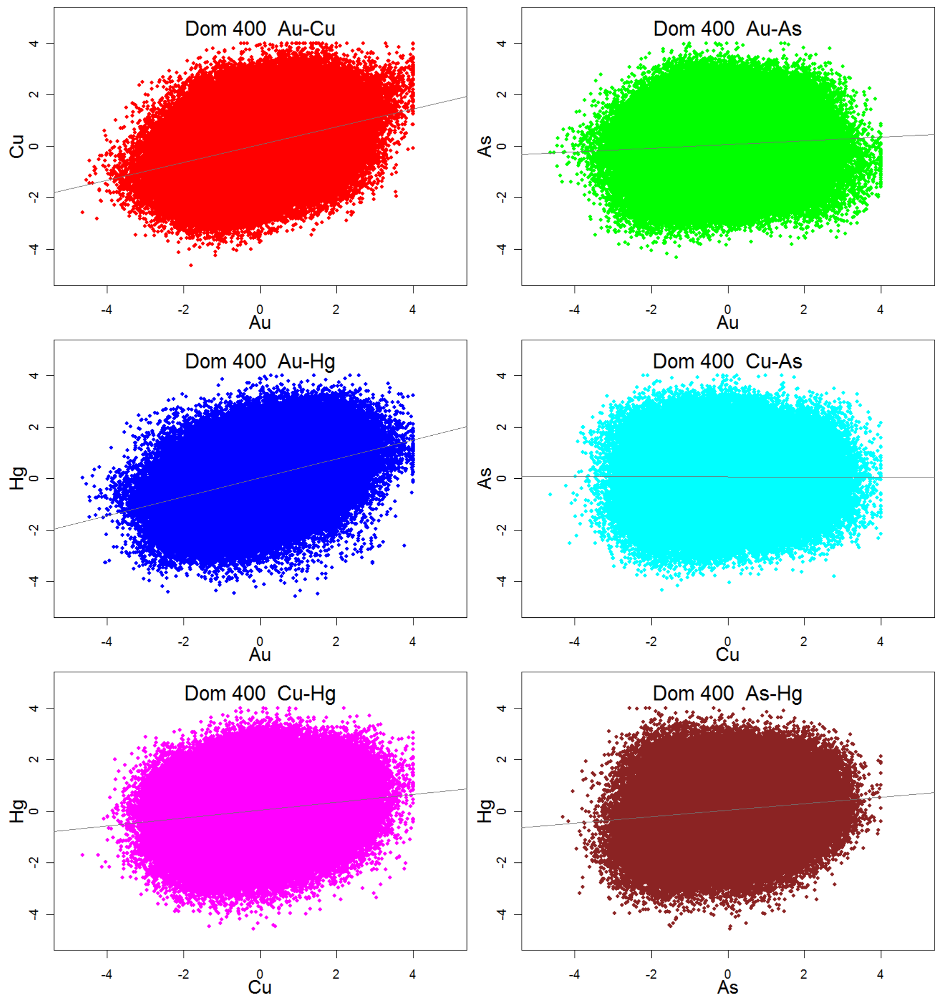

Figure 4. Scatterplots of independent SGS realisations.

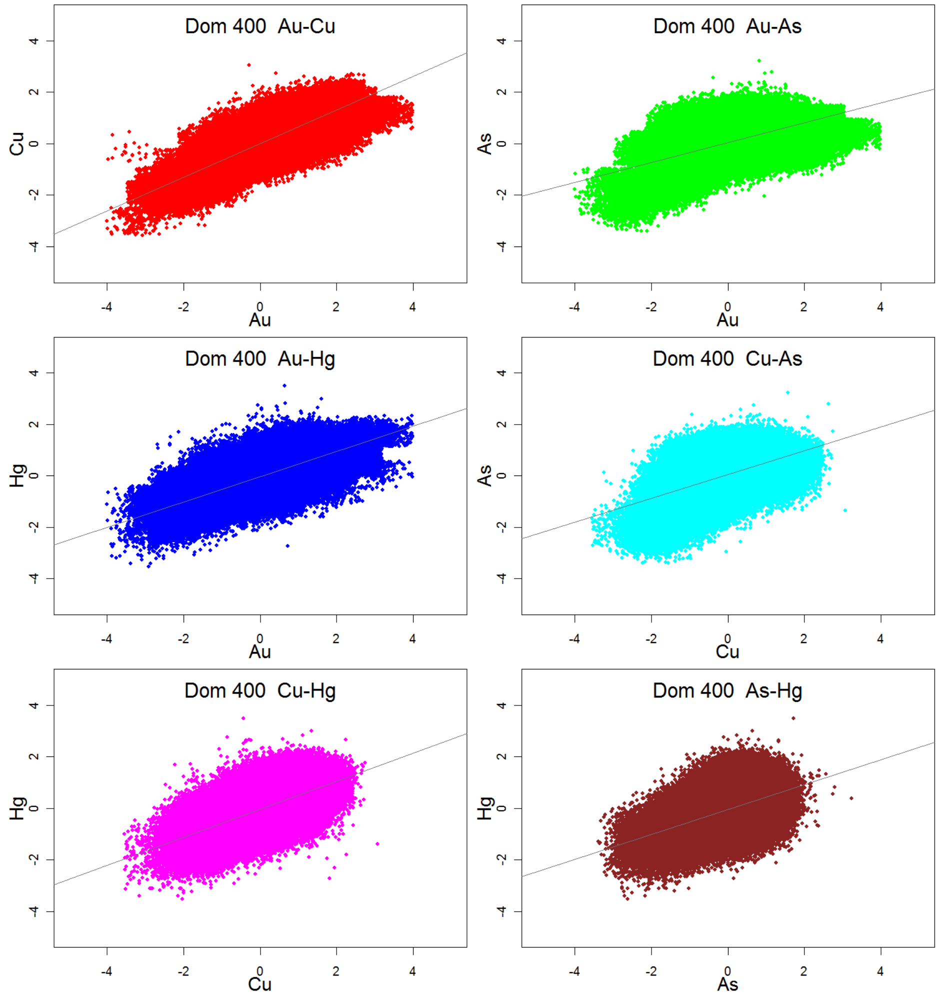

Figure 5. Scatterplots of SGS realisations using local healing based on cloud distributions.

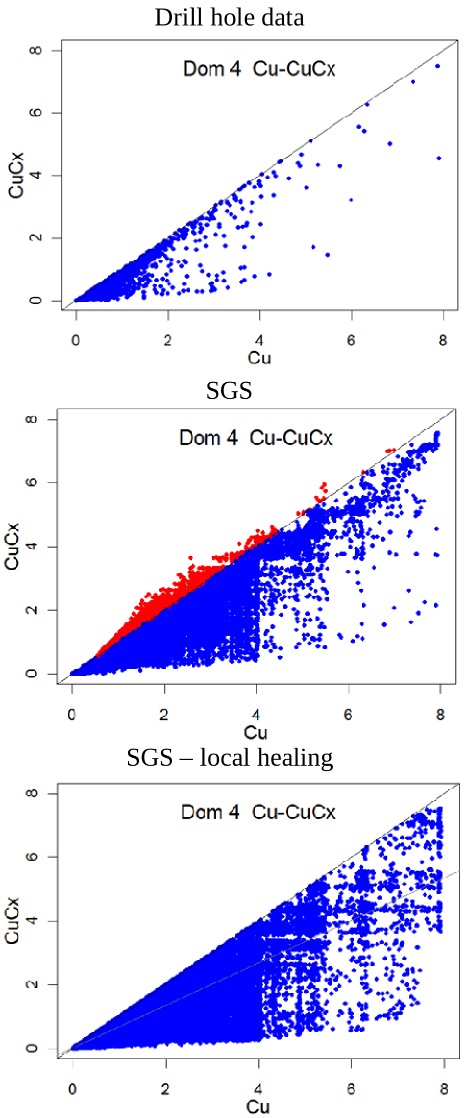

Figure 6. Scatterplots of total copper versus soluble copper.