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EVALUATING CAPITAL INVESTMENT TIMING WITH STOCHASTIC MODELING OF TIME-DEPENDENT VARIABLES IN OPEN PIT OPTIMIZATION

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A new approach to optimizing the timing of capital investment in open pit mines is suggested and demonstrated in an application at a large copper deposit. The approach considers explicitly the uncertain nature of the commodity price cycle and operating costs that can be modelled via stochastic simulation techniques. The stochastic models of prices and costs are fed directly into either a set of nested pits or a direct net present value (NPV) optimization algorithm. This avoids divorcing the delineation of an open mine's pit limit from calculating the related NPV that is common in traditional approaches.

Optimisation, economic evaluation, stochastic simulation, mining schedules

INTRODUCTION

Optimization techniques in open pit mine design have been used for almost 30 years, and form today a well established mining industry standard for all related studies. Implementations of the Lerchs-Grossman algorithm [1], network flow [2], pseudoflow network models [3], mixed integer programming [4 – 7], and others, involve converting a 3D grid of regular blocks representing an orebody to be mined into a payoff matrix by considering mineral grades, as well as economic and mining parameters. These algorithms rely on mining block payoffs to average linearly, but the net present value (NPV) of block payoffs is a non-linear function of the undiscounted block payoffs, and depends on the discount applied to the individual blocks, which in turn depends on the mining schedule. To overcome the issue of discounting block payoffs, traditional implementations of open pit optimization algorithms are designed, using a single estimated orebody model, to find a set of nested open pit limits that maximize the undiscounted financial payoff for a series of constant commodity prices. The maximum NPV open pit limit is then derived by considering alternate mining schedules, usually only the best and worst cases, for each open pit limit. This two-step approach raises three significant issues: (a) separating the open pit limit delineation from the NPV calculation does not guarantee that an optimal (maximum) NPV open pit solution will be found; (b) NPV calculations are based on constant commodity prices and operating costs that fail to consider their time-dependent and uncertain nature; and (c) the single "estimated" ore body model is invariably smoothed, thus it fails to consider short-scale grade variations. This implies that the block model does not accurately reflect the grade and tonnage of ore that will be extracted and processed during mining.

To overcome this inadequacy of undiscounted payoffs, Richmond [8] proposed embedding a scheduling heuristic within an open pit optimization algorithm. This may be seen as an alternative to mixed integer programming approaches [4 – 7] that may become numerically demanding in the case of large deposits. As a consequence, uncertain and time-dependent variables, such as commodity prices and operating costs, can also be incorporated stochastically into the optimization process. This permits strategic options for project timing and staging to be assessed as discrete optimization problems and compared quantitatively, and is more advanced than other recent approaches [9 – 11]. Multiple

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conditional simulations in the optimisation process should, in addition, be considered, so that the mining and financial implications related to small-scale grade variations are honored [[11 – 15]. By considering discounted block payoffs, stochastic models of commodity prices and operating costs, as well as short-scale grade variations, a more accurate discounted payoff matrix (revenue block model) is generated, which in turn will yield an open pit limit that will be closer to the true optimum.

NPV CALCULATIONS WITH UNCERTAIN VARIABLES

Calculation of the NPV for a given open pit limit relies on estimates of numerous parameters, including, but not restricted to, the mineral grades, extraction sequence and timing, mineral recovery, prevailing commodity price, and capital and operating costs. All of these parameters are uncertain and should be modelled stochastically. For example, mineral grade values by geostatistical simulations, operating costs with growth functions, and commodity prices using long-term mean reverting models that account for well known periodicity. Consequently, the cumulative distribution of total financial payoffs for an open pit limit can be derived from the combination of a series of stochastic models of mineral grades, costs, prices, recoveries, etc.

Given L potential NPV outcomes for a block (related to L realisations of grade values, commodity prices, operating costs, etc), we can calculate the NPV for any realisation l:

$$NPV_l = \sum_{j=1}^{B} d^l(b_j) i_j \tag{1}$$

and the expected NPV for L realisations:

$$NPV_L = \frac{1}{L} \left\{ \sum_{l=1}^{L} NPV_l \right\}, \tag{2}$$

where B is the number of blocks under consideration; $d^l(b_j)$ is the discounted value for block b_j for the l realisation; and $i_j = 1$ if b_j falls within the open pit limit and 0 otherwise. The idea is to find the open pit limit that maximizes NPV_L. Additional financial goals, for example minimizing downside risk [16] could also be considered, but are outside the scope of this paper.

ACCOUNTING FOR MULTIPLE ORE BODY MODELS

Pit optimization algorithms found in the literature invariably consider an ore body block model with a single grade value for each block (or parcel). In such an approach, a simple decision rule is used where block b_j is processed using option k if $g_k^l \le z^*(b_j) < g_k^u$, where g_k is the cut-off grade for processing option k (by convention $g_1 = 0$ and $k = 1$ indicates waste), and z^* is the estimated grade value. To account for grade uncertainty in open pit optimization, Richmond [16] proposed incorporating L conditionally simulated grade values for each block. In this approach, multiple grade values $z^l(b_j)$, $l = 1,\ldots, L$ were generated by conditional simulation [17 – 20], and a processing option $k^l(b_j)$ was determined for each realisation. Alternatively, conditional simulation provides short-scale grade variations that permit local ore loss and mining dilution to be readily accounted for in an open pit optimization by [16] (a) generating geometrically irregular dig-lines (that separate ore and waste) based on small-scale grade simulations with a floating circle algorithm; and (b) assimilating the dig-lines into large-scale geometrically regular blocks by a novel re-blocking method. This two-step approach accounts for short-scale grade variation, but also provides “recoverable” grade and tonnage information for large regular blocks suitable for open pit optimization. In other words, the simulated grade models are compressed without loss of accuracy so that optimization is computationally tractable.

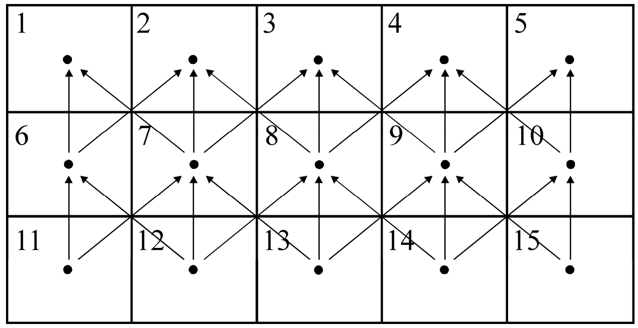

Fig. 1. Directed graph representing two-dimensional vertical ore body model

AN NPV OPEN PIT OPTIMIZATION ALGORITHM

For the vast majority of open pit optimization techniques, a directed graph is superimposed onto the payoff matrix to identify the blocks that constitute an optimal open pit limit. To paraphrase Dowd and Onur [21], each block in the grid, represented by a vertex, is assigned a mass equal to its net expected revenue. The vertices are connected by arcs in such a way that the connections leading from a particular vertex to the surface define the set of vertices (blocks) that must be removed if that vertex (block) is to be mined. A simple 2D example is shown in Fig. 1. Blocks connected by an arc pointing away from the vertex of a block are termed successors of that block, *i.e.* b_i is a successor of b_j if there exists an arc directed from b_j to b_i. In this paper, the set of all successors of b_j will be denoted as Γ_j. For example, in Fig. 1, Γ_8 = [17]. A closure of a directed graph, which consists of a set of blocks *B*, is a set of blocks $B_p \subset B$ such that if $b_j \in B_p$ then $\Gamma_j \in B_p$. For example, in Fig. 1, $B_p = \{1-5, 7-9, 13\}$ is a closure of the directed graph. The value of a closure is the sum of the payoffs of the vertices in the closure. As each closure defines a possible open pit limit, the closure with the maximum value defines the optimal open pit limit.

For simplicity of notation, the algorithm proposed in this paper is described for a single ore body model. The undiscounted payoff matrix $\{w(b), b \in B\}$ typically used for open pit optimization is calculated as:

$$\{w(b) = \text{ton}_b (vz(b) r_k - c_k)\,, \tag{3}$$

where ton_b represents the tonnage of block *b*; *v* is the commodity (attribute *z*) value per concentration unit; r_k is the proportion of the mineral recovered using processing option *k*; and c_k is the mining and processing cost for *k* ($/ton). In practice, r_k and c_k commonly vary spatially, and *v* and c_k temporally. The discounted payoff matrix $\{d(b|S), b \in B\}$, conditional to a mining schedule *S*, that is required for NPV open pit optimization is calculated as:

$$d(b|S) = [\text{ton}_b (v_t z(b) r_k - c_{k,t})]/(1+DR)^t\,, \tag{4}$$

where *t* is the time period in which block *b* is scheduled for extraction and processing; v_t and $c_{k,t}$ are the prevailing commodity price and operating cost at time *t*; and *DR* is the discount rate. In Eq. (4), discounted payoffs are conditional to the mining schedule, as alternate schedules can be derived for the same open pit closure. It is also important to note that cut-off grades, and consequently the processing option *k*, may change in response to commodity price and operating cost fluctuations over time. The traditional floating cone algorithm is a way to decompose the full directed graph problem into a series of independent evaluations of individual Γ_j, and if the sum of the payoffs associated with Γ_j is positive, then b_j is added to B_p. However, a positive undiscounted value for Γ_j does not imply that the discounted value for Γ_j is positive. In other words, negatively-valued successors b_i of block b_j that may be mined significantly earlier in the mining schedule, and receive substantially less discounting, may not be carried by a more heavily discounted positively-valued b_j. Furthermore, the modified schedule may have shifted more profitable b_j into later periods, and additional waste blocks into earlier periods, reducing the discounted value of the pit. As a consequence, NPV optimization with the FCA must consider the directed graph problem globally rather than the traditional independent evaluation of locally decomposed Γ_j. To allow for discounting, Richmond [22] proposed a direct NPV floating cone algorithm (DFC) that proceeds as follows:

1. Select the time for initial investment (start of construction) t_I.
2. Define a cone that satisfies the physical constraints of the desired open pit slope angles.
3. Define an ordered sequence of visiting blocks [1, 2,…, $Б < B$] with positive $w(b)$, by ordering the blocks b_i firstly on decreasing elevation, and then for blocks with identical elevations on decreasing value in $w(b_i)$.
4. Set the open pit closure counter $n = 0$, the initial open pit closure B_P^n to a null set of blocks, and the net present value of initial open pit closure $NPV^n = 0$.
5. Set $j = 0$.
6. Set $j = j + 1$.
7. Float the cone to b_j to create a new closure $B_P^{n+1} = B_P^n + \Gamma_j$ (excluding from Γ_j any block that currently belongs to B_P^n).
8. Determine the schedule S for the new closure B_P^{n+1}.
9. Calculate the discounted payoff matrix $\{d(b|S), b \in B_P^{n+1}\}$ using Eq. (4) and the net present value of the new closure using Eq. (1).
10. Accept the new closure if $NPV^{n+1} - NPV^n > 0$, whereupon the current closure is updated into a new optimal closure, i.e. $n = n+1$, and go to step 5.
11. If $j < Б$, the number of blocks with positive payoffs $w(b)$, then go to step 6.

The version of the floating cone algorithm presented above is heuristic in nature and may not generate an optimal solution. Alternate B_p can be generated by varying the initial investment timing (step 1), the ordered path (step 3), and/or the mining schedule (step 8).

Timing of investments to satisfy corporate constraints, or to take advantage of cyclical commodity prices, can be investigated as mutually exclusive opportunities by varying t_I, which modifies the mining schedule in step 8 above. For example, given a schedule S commencing at $t = 0$, the modified schedule $t' = t + t_I$. For delayed investment, the NPV for many potential production assets will typically be reduced unless maximum production/grade happens to coincide with the peak in cyclical commodity prices. However, for a risk-averse and capital-constrained company, the shift of the capital cost into future years may be strategically advantageous when considered in conjunction with their portfolio of mining assets. Re-initiating the test sequence from the top of the mineral deposit each time a positively valued cone is found and added to the closure is generally regarded to estimate the heuristic maximum undiscounted payoff solution [23]. Computational experimentation on the ordering of blocks in step 3 above suggested that this also holds true for the discounted case when t_I is fixed. Note that, due to re-initiation of the test sequence, it is common for $B_P^{n+1} = B_P^n$ in step 7 above. For such instances, steps 8 – 10 are ignored. Note that, as it is well known, the floating cone algorithm may not return the maximum undiscounted payoff solution. However, Richmond [22] used the algorithm presented above to generate physically feasible solutions; it is also the approach adopted in this study. The author has not investigated whether the Lerchs-Grossman [1] and network flow algorithms [3] could be substituted for the floating cone algorithm, but the non-linearity of the proposed objective function may present some difficulty. The computational efficiency of the proposed algorithm is enhanced significantly when a simple scheduling algorithm in step 8 above is employed. However, more complex risk-based scheduling algorithms, to account for multiple orebody models and production goals [24] could be considered.

APPLICATION AT A COPPER DEPOSIT

This section demonstrates the proposed concepts for a large sub-vertical copper deposit. The geometry and contained copper per level are variable, but there is no strong trend. The options considered in this study were:

- two processing options (ore and waste), i.e. $K = 2$;
- 60 Mt/year mill constraint;
- 25 realizations of copper grades by sequential Gaussian simulation (SGS);
- 25 stochastic simulations of future copper prices with a two factor Pilipovic model that was modified to account for periodicity and cap and collar aversion (Fig. 2);
- 25 stochastic simulations of operating costs with a growth model (Fig. 2);
- monthly copper recoveries randomly drawn from normal distribution with a mean of 80% and a standard deviation of $1\%^2$;
- a fixed annual discount rate of 10 %; and
- initial investment timings at discrete yearly intervals for 5 years.

In this case study, two scenarios are considered: (1) Currently near the peak of the price cycle and higher uncertainty; and (2) currently near the bottom of the price cycle and lower uncertainty. Figure 2 shows 25 stochastic simulations of future copper prices for the two scenarios. The assumptions in this study were: (1) a long-term copper price of \$1.30/lb; (2) the present time \$2.50/lb (Scenario 1) and \$0.70/lb (Scenario 2); (3) an average 8 year copper price cycle; (4) and \$0.50/lb and \$3.00/lb lower and upper aversion values. Note that as time increases, uncertainty in the simulated copper price increases, while the periodicity in the average simulated value and its deviation from the long-term price decreases. The average copper price does not fluctuate symmetrically around the long-term copper price due to the asymmetrical aversion limits. Note that current copper prices and forecast price ranges have not been used, to maintain project confidentiality. Figure 2 also shows 25 stochastic simulations of waste and ore processing costs.

To assess the potential change in NPV against the traditional two-stage pit optimization approach, a base case scenario (\$1.30/lb; 80 % recovery, \$1.90/t waste cost and \$8.50/t milling cost) was run to generate a series of nested pits using a FCA. The E-type (or average) of the 25 SGS realizations was adopted as the single grade model, as it is known to be smoothed. The NPV for this series of pits uses the base case assumptions shown in Fig. 3 as crosses. The maximum NPV under the base case scenario is associated with a pit closure of 26 402 blocks. Note that the capital cost, which could also be modeled stochastically, was not included in this study.

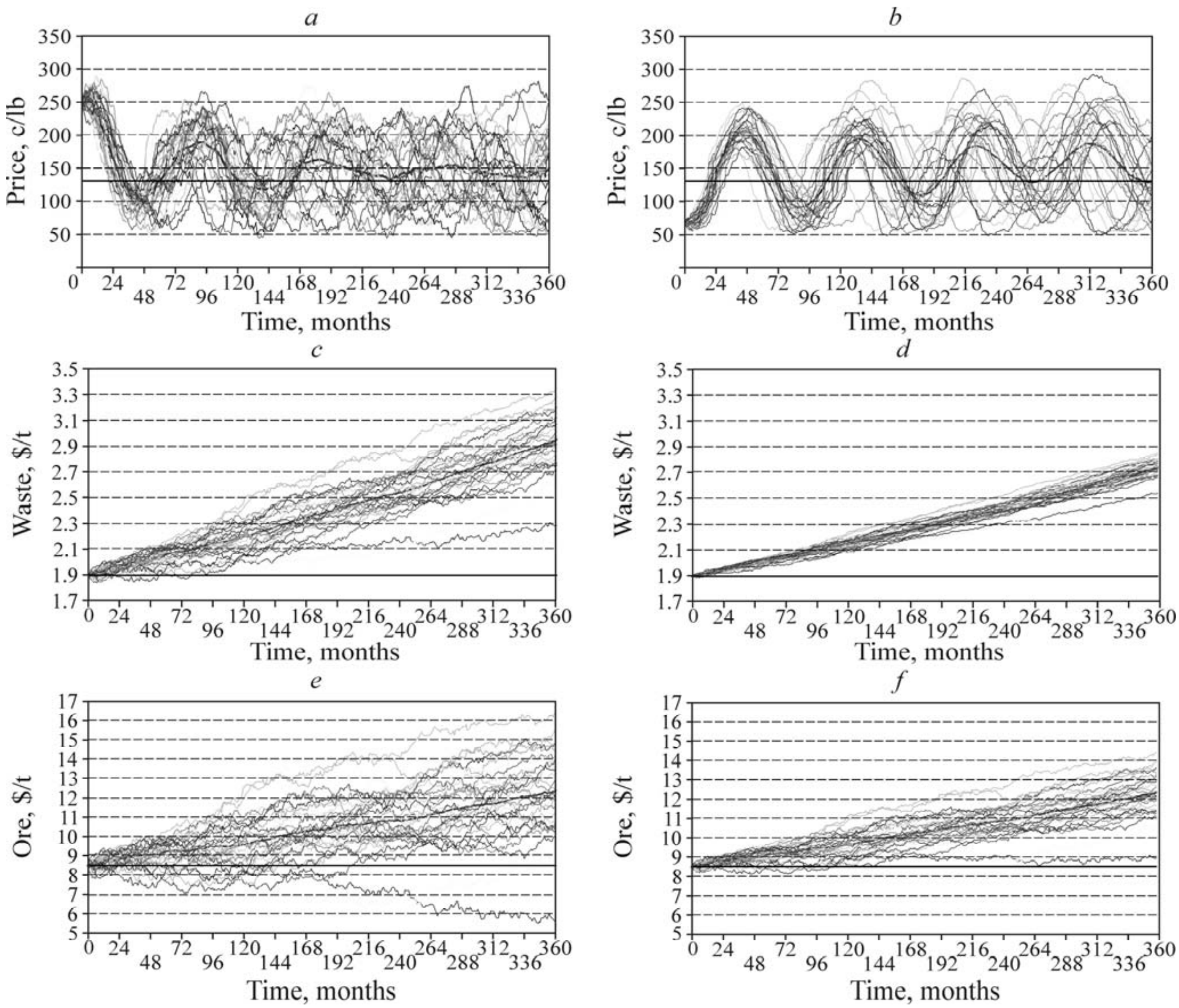

Fig. 2. Stochastic simulations of time dependent variables

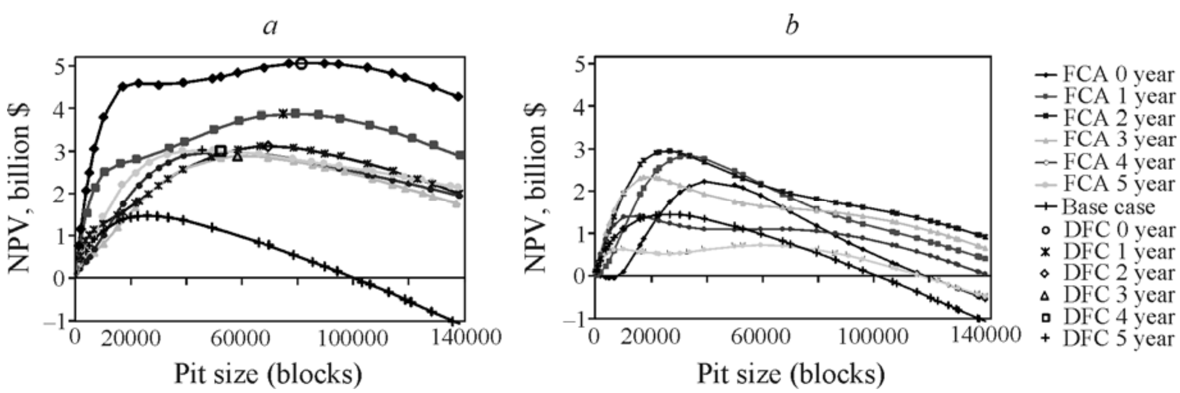

Fig. 3. Pit size versus NPV: FCA — floating cone algorithm; DFC — direct NPV FCA

The NPV for the FCA nested pits was also calculated using the simulated grades, metal prices, costs and recoveries for the six annual investment timings, shown in Fig. 3. Note that (*a*) these curves vary substantially from the base case; (*b*) for Scenario 1, in all instances the maximum NPV pit is significantly larger (49 239 – 85 093 blocks) than the base case and the maximum NPV is higher than for the base case; (*c*) for Scenario 1, delaying the investment from Year 3 to Year 5 results in a higher NPV ($3.02bn versus $2.88bn). At first this relationship may appear counter-intuitive as both costs and discounting are greater, however, it is related to the cyclically higher Cu prices in key production periods; (*d*) for Scenario 2, the maximum NPV pit sizes (15 878 – 38 707 blocks) vary around the base case size (26 402 blocks), but the maximum NPV is higher than for the base case up to Year 3; and, (*e*) for Scenario 2, delaying the investment in Years 1 to 3 all result in a higher NPV than for Year 0.

The NPV of the proposed DFC approach for the six annual investment timings for Scenario 1 is also shown in Fig. 1. Note that considering the mining schedule explicitly in the optimization process meant that it was successful in finding the maximum NPV pit in a single run. Whilst the improvement over the maximum NPV pit from the two-step approach that considered the stochastic inputs was limited (usually <0.5% in NPV), there was often some difference in the pit dimension. It is likely that these differences would be reduced further if additional pit closures had been generated for evaluation in the two-step approach. Computationally, it was more efficient to post process a finite series of pit closures than embed the scheduler in the pit optimization process. In the example shown, the DFC approach that generated a single pit required around the same computational time as that required in generating 36 nested pits by a simple FC approach.

CONCLUSIONS

A novel method for working with discounted payoff matrices during open pit optimization proposed by Richmond [22] was demonstrated. The approach used in this study embedded a simple ore scheduler in a floating cone-based heuristic algorithm. It was a trivial exercise to further consider multiple ore body models, local ore loss and mining dilution, time-dependent commodity prices and costs, and variable metal recoveries during optimization. As a consequence, alternate project development timings could be strategically assessed. Traditional evaluation of a set of nested pit shells with constant metal prices and operating costs failed to determine the maximum NPV pit under uncertain conditions. However, provided that sufficient pit shells were generated and evaluated with the same stochastic price and cost input as for the proposed algorithm, there was little difference in the maximum NPV shell derived. Evaluation of alternate scenarios based mainly on variations in the current price cycle indicate that both the NPV and size of the maximum NPV is strongly influenced by the prevailing point in the price cycle at commencement of the mining operation.

This study demonstrated that uncertainty in future metal prices and operating costs cannot be adequately captured in open pit optimization by simply post-processing a series of nested pit closures with constant values. Stochastic modeling of mineral grades, mineral recovery, commodity prices, and capital and operating costs provide an ideal platform to:

1. Generate an optimal pit to maximise the overall project NPV considering geological and market uncertainty;
2. Determine the optimum investment and project start up timing; and
3. Quantify the multiple aspects of uncertainty in a mine plan.

The example studied in this paper indicates periods of potential financial weakness that could benefit from management focus (e.g. forward-selling strategies, and placing the mine on care and maintenance) prior to difficulties arising.

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